OCR
Oxford Cambridge and RSA

Friday 19 May 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y411/01 Mechanics a

Time allowed: 1 hour 15 minutes

* 9 9 6 7 6 2 8 5 4 9 *

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **Throughout all parts of this question**, the resistance to the motion of a car has magnitude kv^2 N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the car and k is a constant.

At first, the car travels along a straight horizontal road with constant speed $20\,\mathrm{m\,s^{-1}}$. The power developed by the car at this speed is 5000 W.

(a) Show that $k = \frac{5}{8}$. **[3]**

(b) Find the power the car must develop in order to maintain a constant speed of $28\,\mathrm{m\,s^{-1}}$ when travelling along the same horizontal road. **[1]**

The car climbs a hill which is inclined at an angle of 2° to the horizontal. The power developed by the car is 13 000 W, and the car has a constant speed of $20\,\mathrm{m\,s^{-1}}$.

(c) Determine the mass of the car. **[3]**

2 A ball P of mass m kg is held at a height of 12.8 m above a horizontal floor. P is released from rest and rebounds from the floor. After the first bounce, P reaches a maximum height of 5 m above the floor.

Two models, A and B, are suggested for the motion of P.

Model A assumes that air resistance may be neglected.

(a) Determine, according to model A, the coefficient of restitution between P and the floor. **[3]**

Model B assumes that the collision between P and the floor is perfectly elastic, but that work is done against air resistance at a constant rate of E joules per metre.

(b) Show that, according to model B, $E = \frac{39}{89}mg$. **[3]**

(c) Show that both models predict that P will attain the same maximum height after the second bounce. **[4]**

3 The time period T of a satellite in circular orbit around a planet satisfies the equation

$$GMT^2 = 4\pi^2 R^3,$$

where

- G is the universal gravitational constant,
- M is the mass of the planet,
- R is the radius of the orbital circle.

(a) Find the dimensions of G. **[2]**

A student suggests the following formula to model the approach speed between two orbiting bodies.

$$v = kG^{\alpha}c^{\beta}r^{\gamma}m_1 m_2(m_1 + m_2),$$

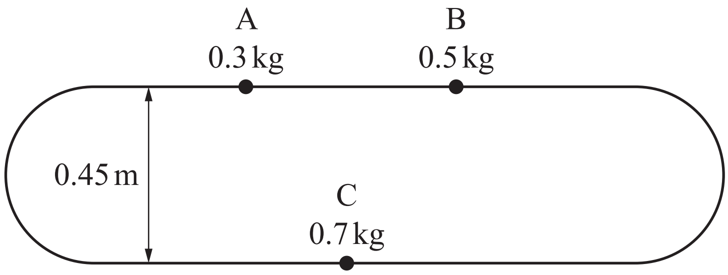
where

- v is the approach speed of the two bodies,
- k is a dimensionless constant,
- c is the speed of light,
- r is the distance between the two bodies,
- m_1 and m_2 are the masses of the bodies.

(b) Use dimensional analysis to determine the values of α, β and γ. **[5]**

(c) Calculate, according to the student's model, how many times greater the approach speed is between a pair of stars which are 6.13 light-years apart and the same pair of stars if they were 8.64 light-years apart. (A light-year is a unit of distance.) **[2]**

4 The diagram shows three beads, A, B and C, of masses 0.3 kg, 0.5 kg and 0.7 kg respectively, threaded onto a smooth wire circuit consisting of two straight and two semi-circular sections. The circuit occupies a **vertical** plane, with the two straight sections horizontal and the upper section 0.45 m directly above the lower section.



Initially, the beads are at rest. A and B are each given an impulse so that they move towards each other, A with a speed of $8\,\text{m s}^{-1}$ and B with a speed of $1.6\,\text{m s}^{-1}$. In the subsequent collision between A and B, A is brought to rest.

(a) Show that the coefficient of restitution between A and B is $\frac{1}{3}$. **[3]**

Bead B next collides with C.

(b) Show that the speed of B before this collision is $4.37\,\text{m s}^{-1}$, correct to **3** significant figures. **[2]**

In this collision between B and C, B is brought to rest.

(c) Determine whether C next collides with A or with B. **[3]**

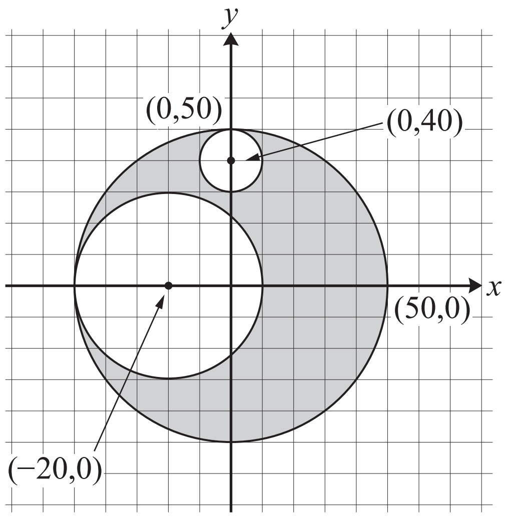
(d) Explain why, if B has a greater mass than C, B could **not** be brought to rest in their collision. **[2]**

5 **Fig. 5.1** shows the uniform cross-section of a solid S which is formed from a cylinder by boring two cylindrical tunnels the entire way through the cylinder. The radius of S is 50 cm, and the two tunnels have radii 10 cm and 30 cm.

The material making up S has uniform density.

Coordinates refer to the axes shown in **Fig. 5.1** and the units are centimetres.

Fig. 5.1

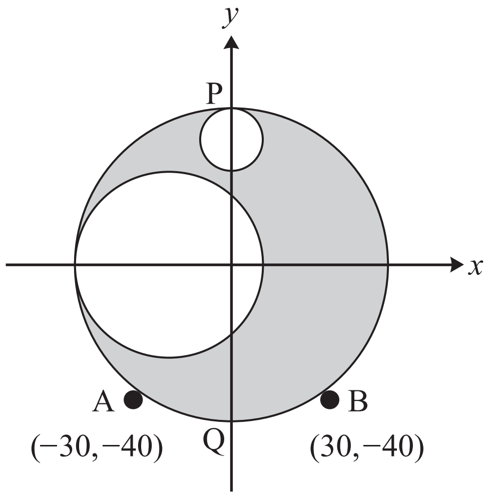


The centre of mass of S is $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x} = 12$ and find the value of $\overline{y}$. **[4]**

Solid S is placed onto two rails, A and B, whose point of contacts with S are at $(-30, -40)$ and $(30, -40)$ as shown in **Fig. 5.2**. Two points, $P(0, 50)$ and $Q(0, -50)$, are marked on **Fig. 5.2**.

Fig. 5.2

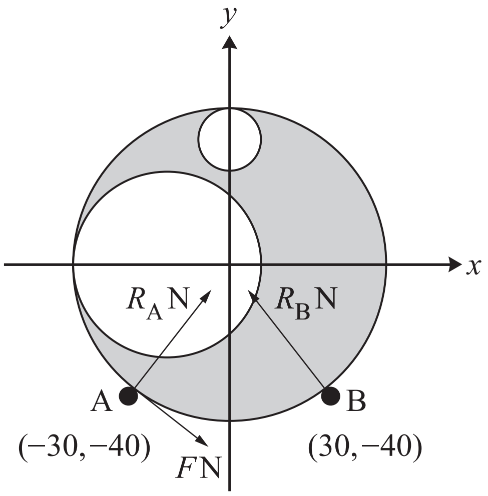


At first, you should assume that the contact between S and the two rails is **smooth**.

(b) Determine the angle PQ makes with the vertical, after S settles into equilibrium. **[2]**

For the remainder of the question, you should assume that the contact between S and A is **rough**, that the contact between S and B is **smooth**, and that S does not move when placed on the rails. **Fig. 5.3** shows **only** the forces exerted on S by the rails. The normal contact forces exerted by A and B on S have magnitude R_A N and R_B N respectively. The frictional force exerted by A on S has magnitude F N.

Fig. 5.3



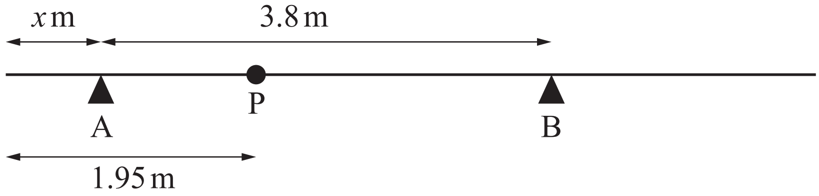
The weight of S is W N.

(c) By taking moments about the origin, express F in the form λW, where λ is a constant to be determined. **[2]**

(d) Given that S is in limiting equilibrium, find the coefficient of friction between A and S. **[5]**

6 A uniform beam of length 6 m and mass 10 kg rests horizontally on two supports A and B, which are 3.8 m apart. A particle P of mass 4 kg is attached 1.95 m from one end of the beam (see **Fig. 6.1**).

Fig. 6.1

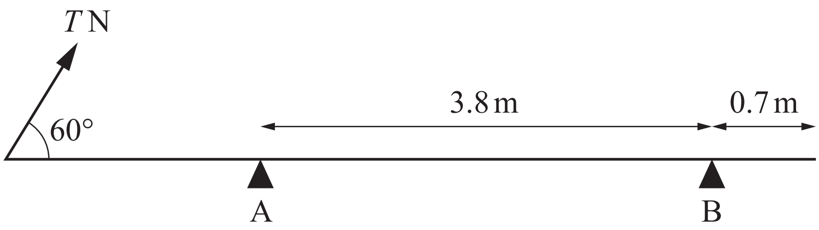


When A is x m from the end of the beam, the supports exert forces of equal magnitude on the beam.

(a) Determine the value of x. **[4]**

P is now removed. The same beam is placed on the supports so that B is 0.7 m from the end of the beam. The supports remain 3.8 m apart (see **Fig. 6.2**).

Fig. 6.2



The contact between A and the beam is smooth. The contact between B and the beam is rough, with coefficient of friction 0.4.

A small force of magnitude T N is applied to one end of the beam. The force acts in the same vertical plane as the beam and the angle the force makes with the beam is 60°.

As T is increased, forces T_L and T_S are defined in the following way.

- T_L is the value of T at which the beam would start lifting, assuming that is not already sliding.
- T_S is the value of T at which the beam would start sliding, assuming that it has not already lifted.

(b) Show that $T_L = 49.1$, correct to **3** significant figures. **[2]**

(c) Determine whether the beam will first slide or lift. **[5]**

END OF QUESTION PAPER